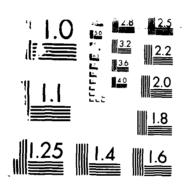
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11 TITLE Include Security Classification)	(11005 )	04.
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Mark J. Ablowitz - Principal Investigator		
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The central theme involved in certain fundamental features associ arising in and motivated by physica work is attested to by the varied a in physics, engineering and mathema wave propagation in a number of are physics, theoretical physics, statimultidimensional solitons, multidim equations, direct linearizations of DBAR problems, Riemann-Hilbert boungeometry, etc.	ated with the nonlinear of problems. The usefuln applications, and wide ar atics. The work accomplies including fluid mechastical mechanics, nonlingersional inverse problems certain nonlinear wave	wave propagation ess of the eas of interest shed involves nics, plasma ear optics, s, Painleve equations,
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Mark J. Ablowitz, Principal Investigator	( <del>315) 268-2375 -</del> 202-71-7-5025	School of Science

## AFOSR-TR. 86-2192

ANNUAL TECHNICAL REPORT

NONLINEAR WAVE PROPAGATION

AFOSR Grant AFOSR-84-0005

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Mark J. Ablowitz

Department of Mathematics

and

Computer Science

Clarkson University

Potsdam, NY 13676



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During the past year significant progress has been made in two related but different areas: (i) Solutions of nonlinear evolution equations (ii) Inverse Scattering. These two fields are bound together via Soliton Theory and the Inverse Scattering Transform.

In (i) the recent progress has been twofold. First a new class of solvable nonlinear singular integro-differential equations has been found. The most interesting of which is the sine-Hilbert equation:

$$Hu_{t} = \sin u$$
 (1)

where  $Hu = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(\xi)}{\xi - x} d\xi$  is the Hilbert transform of u. Equation (1)

generalizes the sine-Gordon equation

$$u_{xt} = \sin u$$
 (2)

to singular integral equations. It should be noted that (1) is the first really interesting solvable singular integral equation since the well known Benjamin-Ono equation (solved by us in 1983):

$$u_t + 2uu_x + Hu_{xx} = 0.$$
 (3)

It should be noted that the Benjamin-Ono equation may be thought of as the singular integral form of the Korteweg-deVries equation

$$u_t + 2uu_x + u_{xxx} = 0.$$
 (4)

A preprint is almost ready on this work. Secondly, we have recently solved an <u>n-dimensional</u> generalization of the sine-Gordon equation which had been studied earlier and derived by a group of differential geometers

at Berkeley: Chern, Terng, Tenenblat. These results are very new and indeed demonstrate that interesting multidimensional nonlinear equations can in fact be solved by inverse scattering. We shall write this work up in the near future. Suffice it to say that it can be expected that the results will be of considerable interest in the mathematics and physics community. It should be remarked that earlier work on solvable multidimensional nonlinear partial differential equations had been confined to three independent variables i.e. "2+1" dimensional problems such as the Kadomstev-Petviashvili, Davey Stewartson and three wave interaction equations. We reported on these results in the recent past.

With regard to (ii) above: Inverse Scattering, progress continues to be made using the so-called  $\bar{a}$  ("DBAR") method. The  $\bar{a}$  method allows us to deal naturally and systematically with multidimensional inverse scattering problems in n dimensions. It leads to formulae to reconstruct local potentials as well as characterization conditions which specify what restrictions on the scattering data are necessary in order to allow reconstruction of a local potential. When restricted to the classical multidimensional time independent Schrödinger scattering problem, our results agree with and indeed go further than those of Faddeev. Moreover, our method applies to the time dependent Schrödinger problem as well as elliptic and hyperbolic systems and is new. We expect that reconstruction/characterization formulae will be able to be found via these methods in other important problems as well e.g. geophysical inverse problems.